



VECTOR (OR CROSS) PRODUCT OF TWO VECTORS



Properties of vector (or cross) product



Angle between two vectors

Lagrange's identity

if $\vec{a}, \vec{b}$ are any two vectors then $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

Area of a parallelogram



Area of a triangle



Question 1.

(i) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$. (NCERT)

(ii) Find the magnitude of $\vec{a}$, where $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

(iii) If $\vec{a} = 3\hat{i} - 5\hat{j}$, $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$, then find $|\vec{a}| : |\vec{b}| : |\vec{c}|$.

Question 2.

- (i) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively when $|\vec{a} \times \vec{b}| = \sqrt{3}$.
- (ii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.
- (iii) If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 3.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. (NCERT)

Question 4.

(i) Find $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$.

(ii) If $|\vec{a}| = 4$; $|\vec{b}| = 2$ and angle between $|\vec{a}|$ and $|\vec{b}|$ is $\frac{\pi}{3}$, find $(\vec{a} \times \vec{b})^2$.

(iii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$ then find $|\vec{a} \cdot \vec{b}|$.

Question 5.

(i) Find the value of λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

(ii) Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

(iii) Find λ and μ $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

(NCERT)

Question 6.

- (i) Find a unit vector perpendicular to the two vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $2\hat{i} + 3\hat{j} + \hat{k}$. (ISC 2004)
- (ii) Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Question 7.

- (i) What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 0$? (NCERT)
- (ii) If either $\vec{a} = 0$ or $\vec{b} = 0$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example. (NCERT)

Question 8.

Find λ such that $\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ are parallel.

Hint: $\vec{a} \times \vec{b} = \vec{0}$

Question 9.

(i) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

(ii) $(\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{i} \times \hat{k}) \cdot \hat{j} + (\hat{i} \times \hat{j}) \cdot \hat{k}$

Question 10.

Find the area of the parallelogram whose adjacent sides are represented by the vectors

(i) $3\hat{i} + \hat{j} + 4\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ (NCERT)

(ii) $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$

Question 13.

(i) If $\vec{a} = 3\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, then find $(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})$.

(ii) Taking $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$ verify that $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$.

$$\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k} \quad \vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$\hat{i}(6+4) - \hat{j}(-4+1) + \hat{k}(8+3) \\ = 10\hat{i} + 3\hat{j} + 11\hat{k}$$

Hence Proved $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

R.H.S $\vec{b} \times \vec{a}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{vmatrix}$$

$$\hat{i}(-4-8) - \hat{j}(-1+4) + \hat{k}(-3-8) \\ -10\hat{i} - 3\hat{j} - 11\hat{k}$$



Question 14.

If θ is the angle between two vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ find $\sin \theta$.

$\vec{a}$ and $\vec{b}$

$$|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|}$$

$$|\vec{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-2+6) - \hat{j}(1-9) + \hat{k}(-2+6)$$

$$\vec{a} \times \vec{b} = 4\hat{i} + 8\hat{j} + 4\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{16 + 64 + 16} = \sqrt{96}$$

$$\sin \theta = \frac{\sqrt{96}}{\sqrt{14} \sqrt{14}}$$

$$\sin \theta = \frac{2\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

$$\begin{array}{r|l} 2 & 96 \\ 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

Question 15.

If $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ then find :

- (i) the sine of the angle between $\vec{a}$ and $\vec{b}$. (NCERT Exemplar)
- (ii) a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$.

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} =$$

$$|\vec{a}| = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{4 + 4 + 16} = \sqrt{24}$$

$$\sin \theta = \frac{\sqrt{384}}{\sqrt{14} \sqrt{24}}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= \hat{i}(4 + 4) - \hat{j}(12 + 4) + \hat{k}(-6 - 2)$$

$$= 8\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\begin{array}{r} 1 \\ 64 \\ 64 \\ \hline 256 \\ 384 \end{array}$$

$$|\vec{a} \times \vec{b}| = \sqrt{64 + 256 + 64} = \sqrt{384}$$

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\hat{n} = \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

$$\vec{a} \times \vec{b} = 8\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\begin{array}{r} 1 \\ 64 \\ 64 \\ \hline 256 \\ 384 \end{array}$$

$$\sqrt{64 + 256 + 64}$$

$$= \frac{8\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{384}}$$

$$\begin{array}{r} 2 \mid 24 \\ 2 \mid 12 \\ 2 \mid 6 \\ 3 \mid 3 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 2 \mid 14 \\ 7 \mid 7 \\ \hline 1 \end{array}$$

Question 16.

(i) Find a vector of magnitude 6 units which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$ (NCERT Exemplar)

(ii) Find a vector of magnitude 19 which is perpendicular to both the vectors $-\hat{j} + \hat{k}$ and $4\hat{i} - \hat{j} + 8\hat{k}$.

$$\hat{n} = \pm 6 \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

Let

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = 4\hat{i} - \hat{j} + 3\hat{k}$$

$$\hat{n} = \pm 6 \frac{(-\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{(-1)^2 + (-2)^2 + (2)^2}}$$

$$= \pm 6 \frac{(-\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} = \pm \frac{6}{3} (-\hat{i} - 2\hat{j} + 2\hat{k}) = \pm 2 (-\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & -1 & 3 \end{vmatrix}$$

$$= \hat{i}(-3+2) - \hat{j}(6-8) + \hat{k}(-2+4)$$

$$= -\hat{i} - 2\hat{j} + 2\hat{k}$$

Question 17.

Find a vector of magnitude 6, perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

Question 18.

If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ find $\vec{a} \times \vec{b}$. Verify that $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other.